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THE GEOMETRY OF DECORATION ON PREHISTORIC PUEBLO POTTERY FROM STARKWEATHER RUIN

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Abstract—This paper contributes further data and analysis to a growing body of literature that use mathematics to enhance interpretation of a culture from styles of its artifacts.

The mathematics employed is the classification of repeating patterns. The artifacts whose patterns are analyzed are specimens of prehistoric Pueblo pottery from Starkweather Ruin in New Mexico. The vessels are housed in the Logan Museum of Anthropology at Beloit College.

The present paper provides:

(a) mathematical background on pattern analysis;
(b) a survey of literature employing these techniques in an anthropological context;
(c) data from observations on the complete corpus of decorated pottery vessels from Starkweather Ruin, a Mogollon-Pueblo site near Reserve, New Mexico;
(d) commentary and analysis.

MATHEMATICAL BACKGROUND

Native and modern peoples throughout the world, from the ancient Egyptians to the twentieth-century Dutch artist Maurits Escher, have experimented in various media with *repeating patterns*. They have investigated the different ways of systematically repeating a basic discrete design element, or *motif*, in patterns along a strip or on a surface. Being systematic means reproducing the basic design motif in the same size by following a uniform rule on where to place each, based on considerations of symmetry. Creative artisans in each culture must have come up against the strong limitations that, as we will see, are imposed by being systematic in this sense. [In the definitive technical terminology of Grünbaum and Shephard [1, p. 165; 2, pp. 204–205], we are restricting consideration to *discrete non-trivial* (*monomotif*) *patterns*.]

The commonly accepted notions of symmetry in two dimensions are susceptible to analysis into four basic geometrical elements:

(1) *reflection* of the motif across a straight line, producing a mirror-image, characteristic of so-called bilateral symmetry;
(2) *translation*, or repitition at regular intervals, of the motif in a straight line;
(3) *rotation* of the motif about a center, so that it repeats at regular angular intervals;
(4) *glide reflection*, a combination of translation followed by reflection, best illustrated by the pattern produced by a person's footprints.

These basic elements are called *symmetries*. Synonymous mathematical terms are *rigid motions*, *congruence transformations*, *distance-preserving maps* and *isometries* (from the Greek for "same size").

We say that a pattern possesses a particular symmetry if the motion of that symmetry, when applied to the pattern as a whole, takes every exemplar of the motif into another one exactly. The collection of symmetries that a pattern possesses is called its *symmetry group*, where "group" is used in the technical mathematical sense to refer to fundamental ways in which the symmetries interact algebraically. For example, performing one symmetry motion, followed by a second one, must result in a combined motion that is a symmetry already in the collection.

Patterns may be classified by their symmetry groups. If two symmetry groups are the same (*isomorphic*), then the patterns they represent are of the same (*symmetry*) class. Grünbaum and Shephard [1, p. 164; 2, pp. 38–40] give the technical mathematical details: two symmetry groups are of the *same class* if one can be transformed into the other via conjugation by an affine transformation of the plane. For monochromatic patterns this condition is equivalent to just the existence of a group isomorphism between the symmetry groups, as Schwarzenberger [3, pp. 12–13] notes.

The investigation below is based upon symmetry group classification. Finer classifications, by *henomeric type* and by *diffeomeric type*, are introduced by Grünbaum and Shephard [1, pp. 17; 2, p. 220]. The quantity of data available to us does not warrant our using a finer classification, although archaeologists should be aware that Grünbaum and Shephard's investigation have revolutionized the field of pattern analysis by providing clarification and systematic classification of concepts.

We first consider *monochromatic* patterns, those in which all exemplars of the motif are executed in a single color against a background of a different color.

Patterns can be classified by the number of directions in which they admit translation.

Finite patterns allow no translations (nor glide reflections) and hence are limited in their symmetry elements to rotations about a single point, plus possibly reflections as well. Their symmetry groups of finite patterns are called *point groups*. There are two families: the *dihedral* groups, which have reflections, and which characterize the symmetries of flowers with bilaterally symmetric petals; and the *cyclic* groups, with no reflection symmetries, which characterize the symmetries of pinwheels or flowers whose petals do not have bilateral symmetry. The notation we will use for these are dn for the former and cn for the latter, where n is the number of petals. For example, $d4$ and $c4$ are the groups of symmetries of a four-leaf clover and a swastika, respectively. There is no restriction on the conceivable number of petals or rotation arms, so that there is an infinite number of groups in each of the two families. Thus, even after the motif and the colors for figure and background have been selected, there is still very wide (if monotonous) latitude for the artisan in choosing a pattern to execute the work.

One-dimensional patterns, those in which translations are allowed along a single axis, are variously referred to as *frieze patterns*, *strip patterns*, or *band patterns*. Here a very stark limitation asserts itself. Mathematical analysis [2, p. 218] shows that *there are only seven classes of strip patterns*! That is, once the basic motif is selected, and the colors of figure and background specified, there are seven ways to repeat the motif systematically. Several notations have been devised for these seven symmetry groups; with a special concern for easy adaptability of the notation to patterns with more colors, we have elected to use the notation of the *International Crystallographic Union* [4]. The left part of Fig. 1 shows samples of each monochromatic pattern, together with the notation. The basic motif for all the sample patterns is an asymmetric right triangle. Crowe and Washburn [5] give a similar table that cross-references notations of several authors.

The international notation succinctly summarizes the symmetries of the pattern. The full notation for a monochromatic strip pattern is made up of four symbols:

(a) The first is always a *p* (for "primitive"), indicating that every symmetry moves every motif exemplar.

(b) The second symbol is an *m* (for "mirror") if the pattern has vertical reflection lines, that is, reflection symmetries perpendicular to the direction of the pattern. The symbol is a *1* if no such symmetry is present.

(c) The third symbol is an *m* if the central axis along the length of the pattern is a reflection line, and an *a* if it is a line along which glide reflection takes place without mirror reflection being present. Again, a *1* symbolizes lack of symmetry.

(d) The fourth symbol is a *2* if the pattern had two-fold (i.e., half turn, or 180°) rotation symmetries. Otherwise the symbol is a *1*.

Crowe [6] and Zaslow [7] give further practical details on how to classify strip patterns.

Patterns that repeat in more than one direction on a two-dimensional surface are called *repeating plane patterns*, *periodic patterns*, or *wallpaper patterns*. There are only 17 classes of periodic patterns, shown in Fig. 2. Since these patterns figure only occasionally in the Pueblo pottery to be discussed later, we refer the reader to Zaslow [7] and Schattschneider [8] for discussion of notation, classification, and recognition of these pattern classes.

Three-dimensional symmetry groups describe placement of atoms in crystals, and crystallography was the motivation for their enumeration in the late nineteenth century. There are 230 such groups, called *Fedorov groups*. Mathematicians and crystallographers have also enumerated the symmetry groups in four and higher dimensions—see Schwarzenberger [3, pp. 132–135] for historical remarks.

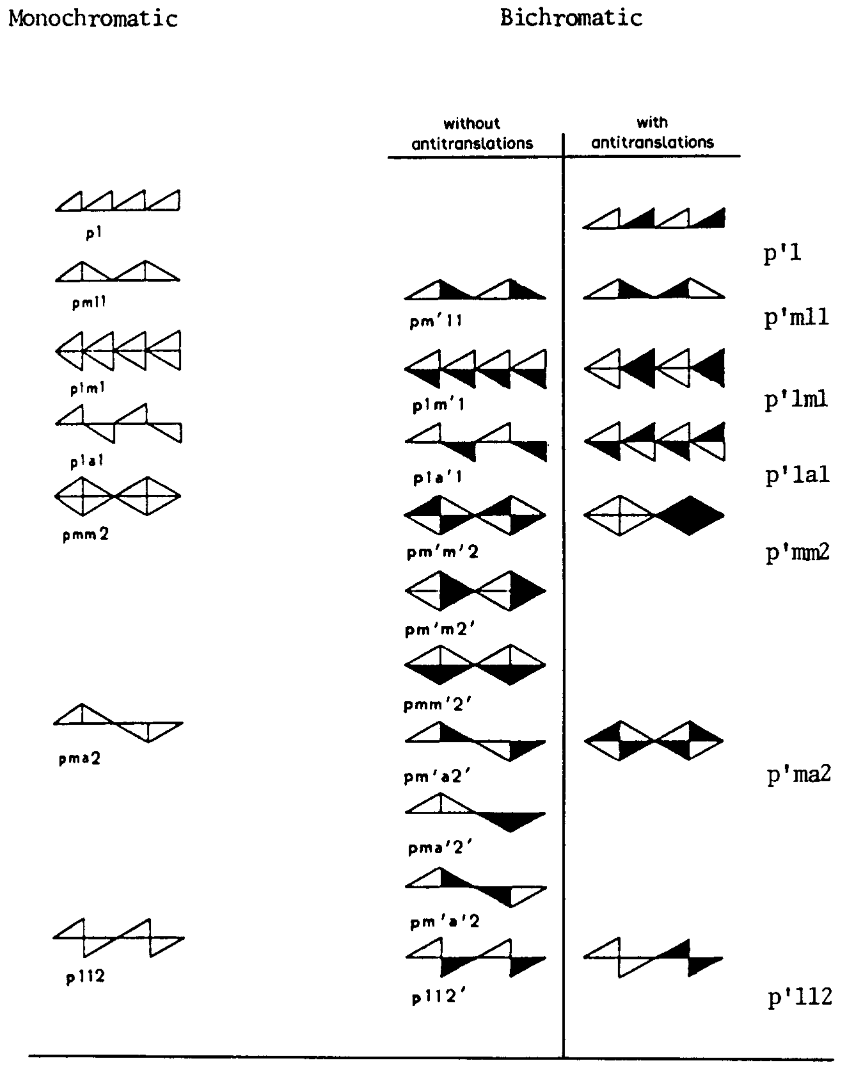

Fig. 1. The seven monochromatic and 17 bichromatic strip pattern classes, with international notation. (Adapted from Shubnikov and Koptsik [34, Fig. 208, p. 271].)

For patterns in which the motif is executed in more than one color, the scheme of color repetition must be systematic; and this means that the symmetries of the uncolored pattern systematically permute the colors. Of interest in connection with Pueblo pottery are the *bichromatic* patterns, in which the motif appears in two different colors, or in two colors against a neutral background that is either uncolored (such as an unpainted pot surface) or of a third color. (Note that Lockwood and Macmillan [9] use the variant term dichromatic to embrace all of: uncolored patterns, "particoloured" (our bichromatic) patterns, and "grey" patterns.)

It is important to note that so-called "black-on-white" pottery will generally have a monochromatic pattern, because the motif occurs only in black; while some "polychrome" pottery, such as red-and-black-on-cream, will have a bichromatic pattern, based on the two colors in which the motif appears. In counting the number of colors involved in the symmetry, it is necessary to decide whether all of the colors on the object embody the motif, whether the background itself changes color, or whether one color functions as a neutral background for the others.

Some Starkweather "black-on-white" pots feature a cross-hatching that we have decided to consider as a distinct color, so that the two colors black and hatched appear against a white background. The patterns are consequently analyzed as bichromatic.

Each of the monochromatic symmetry groups discussed above gives rise to one or more bichromatic symmetry groups, depending on how many different ways the color change can interact

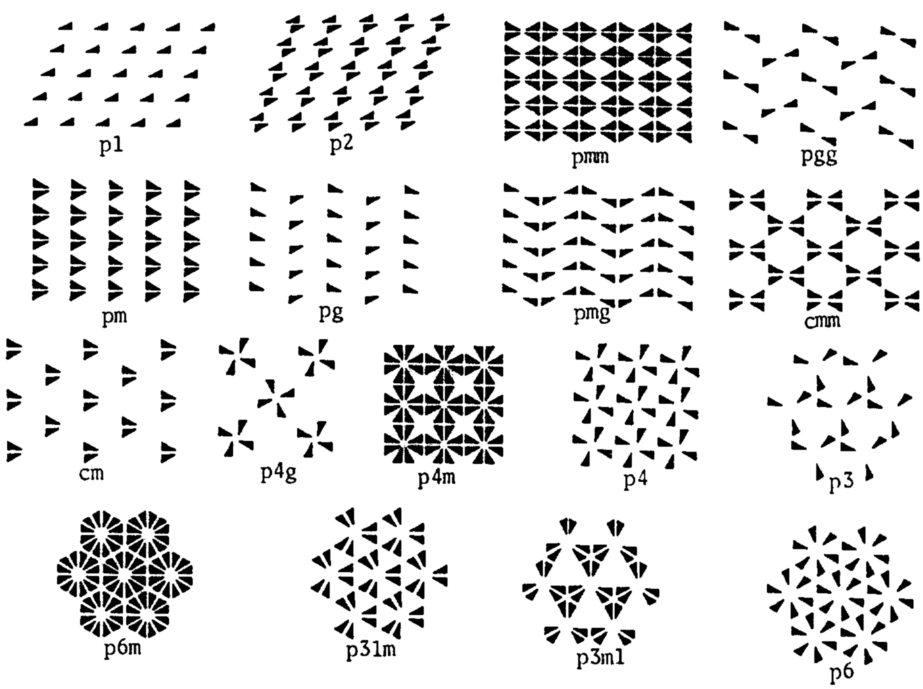

Fig. 2. The 17 monochromatic periodic pattern classes. (Adapted from Shubnikov and Koptsik [34, Fig. 150, p. 157], which in turn was adapted from M. J. Buerger, *Elementary Crystallography* (rev. edn). Wiley, New York (1963).)

with the symmetries of the uncolored pattern. The rotation symmetry of the cyclic *point groups* can either preserve color (giving back the original monochromatic pattern), or else change color, thereby producing one new infinite class of patterns. (The color change only "works out" if n is even.) The dihedral point groups offer color the opportunity of interacting with either or both of the rotation and reflection symmetries, resulting in two new infinite classes patterns.

There are a total of 17 *bichromatic strip groups*, shown with their notation on the right of Fig.1. Translations that reverse color are called *antitranslations*. For these we agree with Crowe and Washburn [5] in deviating slightly from the international notation of Fig. 1, replacing p_a and $p_{a'}$ by p' for typographical reasons. Crowe and Washburn further describe the pattern classes and give a guide and flowchart for their recognition.

At variance with this result, both Washburn [10] and Macdonald and Street [11] arrive at a total of 21 bichromatic strip symmetry patterns. Not surprisingly, their analyses are based on a different criterion for which patterns should be considered distinct. Jarratt and Schwarzenberger [12] explain: "... [Macdonald and Street] begin by selecting a representative pattern corresponding to each of the seven uncoloured frieze groups and then colour the patterns. Resulting coloured frieze patterns are considered equivalent if they can be superimposed on one another (modulo permutations of the colours)." As in Jarratt and Schwarzenberger [12] and Crowe and Washburn [5], the analysis of this paper is based on the criterion of symmetry class and the resulting 17 pattern classes. The differences between the two criteria are discussed with examples in a later section of this paper. examples in a later section of this paper.

The 17 two-dimensional symmetry groups produce 46 *bichromatic periodic groups*—see Lockwood and Macmillan [9, pp. 63–66, 198–202] and Shubnikov *et al.* [13, Fig. 144 opposite p. 220]—and the 230 three-dimensional Fedorov groups produce 1651 bichromatic groups in three dimensions, called *Shubnikov groups*—see Shubnikov *et al.* [13, pp. 175–201].

Lockwood and Macmillan [9, pp. 67–70] remark that a polychromatic period pattern having rotation can have only three, four or six colors; and they illustrate all 11 such patterns. Again, their observation is peculiar to their definitions, which are based on a very restricted view of color symmetry. Jarratt and Schwarzenberger [12], Senechal [14] and Wieting [15] use the more-conventional definitions to arrive at 96 four-colored groups (Wieting illustrates them all), and a total of more than 900 groups for two to 15 colors. Other authors suggesting a more restricted view are Macdonald and Street [11, 16] and van der Waerden and Burckhardt [17]. Loeb [18] imposes restrictions springing from crystallographic considerations.

Schwarzenberger [19] gives a definitive overview of color symmetry which dispels most of the confusion that has beset the subject. All investigators agree completely in the case of two-color patterns.

Despite the above remarks, it may still be surprising that beyond bichromatic coloring, increasing the number of colors does *not* vastly increase the number of pattern classes of *strip* patterns. Jarrett and Schwarzenberger [12] show that the number of strip pattern classes with N colors is 7 when N is odd, 17 when N is divisible by 2 but not by 4, and 19 when N is divisible by 4. (Lockwood and Macmillan [9, p. 68] claim that polychromatic strip patterns can be based only on the uncolored classes *p111*, *p1m1* and *p1a1*; their observation is again based on a narrower definition of a pattern.)

LITERATURE SURVEY

Washburn [10, pp. 11–12] enumerates some of the few papers that have used symmetry patterns to classify and analyze designs. Several others have appeared, however, and we find it useful to survey chronologically all of the contributions, especially since some omitted by Washburn concern artifacts produced by native peoples of the Americas.

Speiser [20] enumerated the monochromatic point, strip and periodic pattern classes and offered illustrations from ancient Egyptian ornamentation as collected by Jones [21]. Woods [22] presented a mathematical enumeration of the point, strip and periodic patterns classes, including all uncolored and bichromatic symmetry groups. Buerger and Lukesh [23] correlated the common knowledge of chemists about symmetry groups in crystallography with ornamentation in two dimensions. Their article may be the origin of the name "wallpaper groups".

Stafford [24] analyzed the colored repeating patterns on the amazingly beautiful embroideries produced around 200 B.C. by the Paracas culture of Peru. It appears she was unaware of the mathematical analysis of symmetries. Although the initial work on uncolored patterns had been done in the late nineteenth century by crystallographers, had been exposited in Woods [22], and had found its way into mathematical books such as Birkhoff [25] and Hilbert and Cohn-Vossen [26, pp. 56–88], it was certainly not common knowledge, even among mathematicians. Color symmetry was first investigated only in the 1940s and 1950s. The Paracas embroideries feature mainly translations and glide reflections with color changes. Irregularities, instances of "broken" or unbalanced symmetry, are common.

Brainerd [27] suggested analyzing ceramic designs according to symmetry properties. Mueller [28, 29] applied the theory of one- and two-dimensional monochromatic symmetry groups to analyze the Moorish patterned ornamentation on the walls of the Alhambra in Spain. Shepard [30] used the seven monochromatic strip groups to classify strip patterns on pottery from the American Southwest. Garrido [31] used the monochromatic strip and period groups to classify ornaments on monuments of ancient Mexico. He noted relative percentages of occurrence of each pattern class, pooling together monuments from the wide diversity of Mexican cultures.

MacGillavry [32] presented examples from the work of the Dutch graphic artist M.C. Escher (1898–1971) to illustrate each of the 17 wallpaper pattern classes, as well as some of the color pattern classes. Escher made several prints specifically for the volume, in order to fill out the catalogue. It is interesting to observe that Escher attributed his initial interest in his intertwining figures to a visit to the Alhambra in 1935–1936; he felt that the patterns of the Moors could be very greatly enriched by using animate figures for motifs, a practice forbidden in Islam. Also worthy of note is that Escher's half-brother was a chemist, who no doubt was aware of the use of symmetry groups in chemistry.

Rappoport [33] was referred to by Shubnikov and Koptsik [34], but we have not seen the former work.

In 1971 Donald W. Crowe began a series of articles [6, 35–38] analyzing monochromatic symmetry patterns in African artifacts, including Bakuba decoration (Zaïre), Benin bronzes (Nigeria), and Begho pipes (Ghana). This work has been furthered by Zaslavsky [39–41].

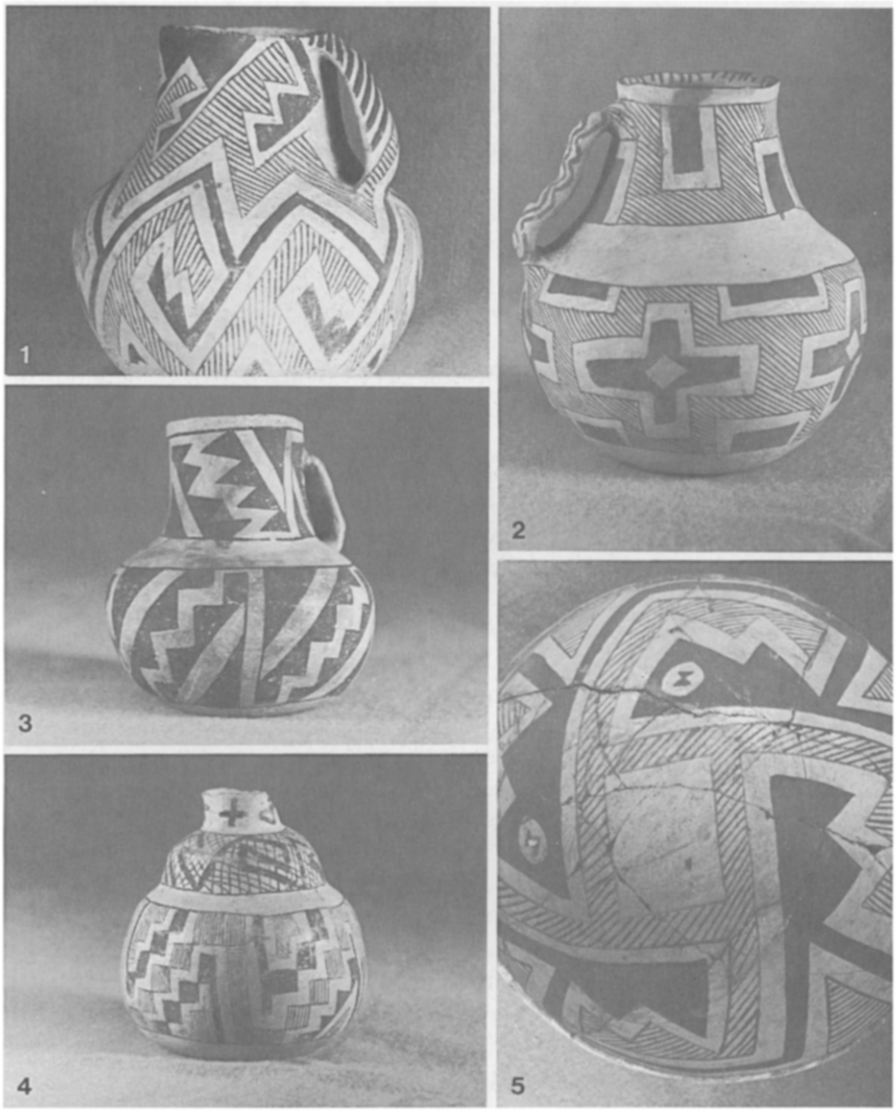

Plate 1. 22399 Reserve pitcher. Neck has six elements, motif 19 and pattern *p112'*. Body has seven elements, motif 16 and pattern *p112'*. Body design continues across handle.

Plate 2. 22427 Reserve pitcher. Neck has six elements, a bar motif and pattern *pma2*. Body has five elements, a cross and bar motif and pattern *pmm2*. Handle has five elements, a wavy line motif and pattern *pm11*.

Plate 3. 22403 Reserve pitcher. Neck has four elements, motif 36R and pattern *p112*. Body has five elements, motif 36 and pattern 36. Handle has two elements, motif 73 and pattern *c2*. Body and neck patterns are mirror images.

Plate 4. 22409 Reserve eccentric olla (pitcher). Neck has five elements, cross motif and no pattern. Lower neck has six elements, motifs 6 and 41 and no pattern. Body has six elements, motifs 19, 46R and 47R, and pattern *p112* (with deviation from exact symmetry).

Plate 5. 22423 Tularosa bowl. Interior has four elements, motif 19 and pattern *p112'* (with deviation from exact symmetry).

Recent years have seen notable further attempts to arouse interest among anthropologists in applying symmetry techniques to analysis of patterns on artifacts. Zaslow and Dittert [42–44] focused on decorations on ceramics of the Hohokam culture, largely from the site at Snaketown, Arizona. Many examples of this pottery bear two-dimensional patterns. A major feature of Zaslow and Dittert [43] is an effort to detect an evolutionary development in Hohokam design. The inferred development parallels established chronology and even suggests some refinements of the sequence. The authors maintain that "cultural continuity is implied by the continuity in pattern development"

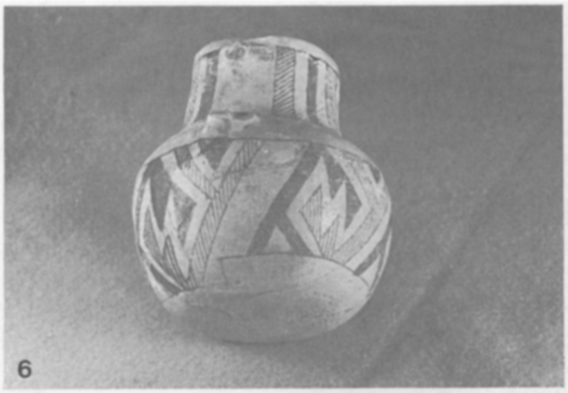
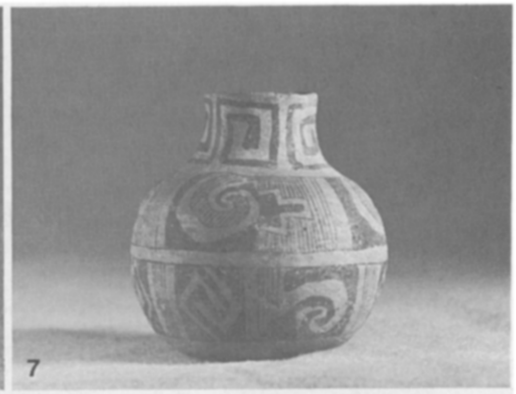


Plate 6. 22387 Tularosa pitcher. Neck has seven plus elements, shaded bars and pattern *p112* (with deviation from exact symmetry). Body has five elements, motif 16 and pattern *p112'* (with deviation from exact symmetry). Missing doube (?) handle, right-angle jump in border, "probably Starkweather" (Nesbitt).

Plate 7. 22395 Jar. Neck has five elements, motif 71, and pattern *pma2* (with deviation from exact symmetry). Upper body has four elements, motif 37 and pattern *p111*. Lower body has seven elements, motifs 53 and 73 and pattern *c2*. Additional symmetry element of curved vs jagged.

Plate 8. Representative ceramics from Starkweather Ruin.

Plate 9. 22398 Tularosa pitcher. Rim has 17 elements, motif 25, and pattern *pm11* (with deviation from exact symmetry). Body has three elements, motifs 52 and 53, and pattern *p'112* (with deviation from exact symmetry). Handle has three elements, wavy line motif, and pattern *p112*.

Plate 10. 22435 Jar. Part of neck and most of body missing. Body has motif 16 and pattern *p2'*.

(p. 25) and tentatively suggest "a connection between social factors and the pattern class selected for ceramic decoration" (p. 26).

In addition, Zaslow, a chemist, offered in Ref. [7] a copiously illustrated guide to employing one- and two-dimension monochromatic symmetry groups in analyzing ceramic decorations. Worthy of special note is a section on building two-dimensional patterns by placing horizontal strips adjacent to one another (pp. 27–36). In Ref. [45] he concentrated on handedness ("mirror orientation") of motif elements as an important component of pattern development and indicator of chronology.

Zaslow [46] identified "shared pattern features" in Hohokam ceramics and Oaxaca Valley mosaic panels, concluding that close similarities preclude independent development. Zaslow and Lindauer [47] used geometric patterns on ceramics to infer an Anasazi influence on Hohokam ceramics.

The papers confined themselves to monochromatic patterns, as had all previous authors except Woods [22] and Stafford [24]. As a result, Fig. 11 (p. 25) of Zaslow [7], classified *p2* there, should be the bichromatic pattern p'_b2, in the notation of Lockwood and Macmillan [9]; and Fig. 14 (p. 30), classified *p4gm*, may be viewed as $p'4gm$ (p'_c4gm, in the notation of Crowe and Washburn [5]) if one considers the shadings as one color, and as a much more complicated color group if the shadings are considered two different colors.

The major feature of Washburn [10] was consideration of bichromatic patterns. Washburn classified designs on 152 whole vessels, 80% from a single site at Mariana Mesa, and the others from sites within 5 miles of it. Following Woods [22], she referred to reversal of the two colors under a symmetry as the process of *counterchange*. Washburn made a finer distinction for two-dimensional patterns, so that she regarded as different two designs whose appearance changes according to alignment of the design vertically or horizontally [10, pp. 26–27].

This gave her a total of 23 monochromatic two-dimensional patterns based on a fixed orientation. We might call them the 23 *oriented* wallpaper patterns, because the differences in appearance trace to differences in orientation of the motif. Washburn distinguished between designs with the same pattern but aligned at 90° to each other. This suggests the further possible refinement of also distinguishing between designs with the same pattern but different *handedness* of the motif, or aligned at 180° ("upside down") or 270° to each other.

Such distinctions cannot apply to monochromatic strip patterns, so she still had seven of those. However, in enumerating bichromatic strip patterns she arrived at a total of 21 (instead of 17). Here the expanded number is due not to fixed orientation of the strip, but to grouping of motif copies into "units." Though her illustrations [10, Fig. 19, p. 26] suggest to the reader that the grouping is based on proximity of motifs, in fact the basis for grouping is not explained. However, Macdonald and Street [16] also arrived at the number 21.

Examples are easily constructed that support Washburn's contention that one *should* distinguish two designs whose symmetry groups are the same, and which at the same time defy the ability of her system of classification to do so. In our Figs 3(a) and (b) we use thick dark lines to represent one color, thin light ones to represent the other. The strips depicted have the same symmetry group $p'mm2$ and the same motif, a line segment at 45° to the horizontal. One strip appears to be made up of Xs and the other of lozenges.

Washburn's analysis was based on choosing a basic "unit" of the pattern, possibly larger than the fundamental motif. If we regard a single-color X and a single-color lozenge as the respective "units" of the two patterns, we classify both as $1\text{–}2_2 11_2$, in Washburn's notation. On the other hand, if we regard a bicolor X and a bicolor lozenge as the respective "units," then we arrive at her different class $1\text{–}2^2 11^2$ for what are precisely the same strips as before.

The impact of the argument above is made stronger by coloring parts of the patterns, producing one strip made of single-color bow ties—or is it made of two-color diamonds [Fig. 3(c)]?—and another made of two-color bow ties—or is it made of single-color diamonds [Fig. 3(d)]?

Washburn's distinctions also led her to a correspondingly large number for the "counterchanged" wallpaper patterns than the traditional 46 bichromatic ones.

Apart from the incorporation of counterchange, the great contribution of Washburn's book was the theoretical background (pp. 3–10) she offered for the establishment of symmetry analysis as an important tool of the anthropologist. She has continued to use symmetry considerations in studying ceramics [48, 49].

Schattschneider [8] offered examples of Chinese lattices in 14 of the 17 monochromatic pattern classes, while Niman and Norman [50] recounted a classroom activity based on identifying pattern classes in Islamic art. Rose and Stafford [51] outlined a possible course in the mathematics of symmetry, providing identification algorithms for pattern classes. Crowe and Washburn [52] provided flowcharts for identifying the monochromatic and bichromatic pattern classes. Ascher and Ascher [53] noted a correspondence between the symmetries of designs on Inca pottery and the organization of the Inca quipu recording system. Crowe and Washburn [5] analyzed the bichromatic patterns present on nineteenth-century pottery of San Ildefonso Pueblo, a village on the Rio Grande between

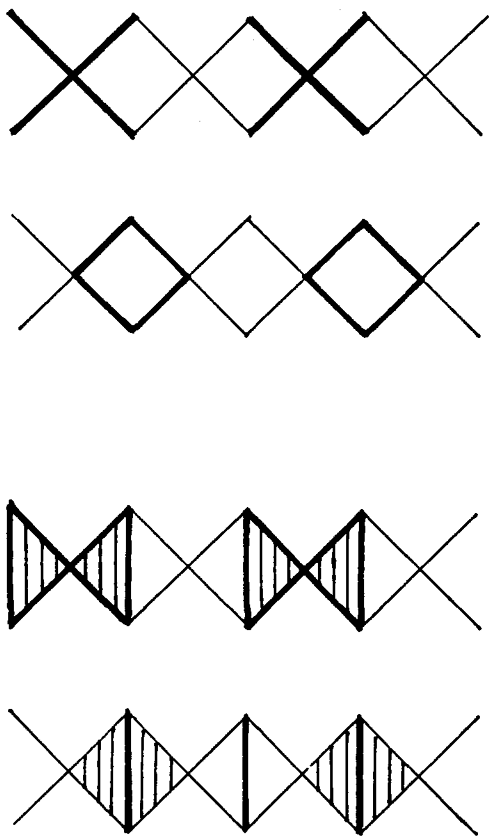

Fig. 3. Patterns of the same symmetry class distinguished [(a) from (b), (c) from (d)] under the classification systems of Washburn [10] and MacDonald and Street [11].

Los Alamos and Santa Fe, New Mexico. This pottery represented a revival for sale to tourists and exhibits a Spanish floral influence. Crowe and Washburn worked from illustrations in Chapman [54]. They showed that 14 of the 17 bichromatic patterns occur, and they offered comparisons with older San Ildefonso pottery styles, concluding that changes in pottery patterns exhibit cultural change.

A comprehensive treatment of analysis of plane patterns on cultural artifacts was given by Washburn and Crowe [55] and a tutorial module was published by Crowe [72].

THE STARKWEATHER SITE

We turn now to a description of Starkweather Ruin and analysis of the symmetries on the pots recovered from it. Excavation at the site was undertaken during the seasons of 1935 and 1936 by Paul H. Nesbitt, professor of anthropology at Beloit College, in Beloit, Wisconsin. Nesbitt was accompanied by undergraduates from the College, thereby continuing a strong tradition of involving students in research. Beloit students have participated in excavations in many parts of the world; and the Logan Museum on the campus contains many artifacts from those expeditions, in displays created by students.

Readers desiring a concise introduction to the relavant peoples, cultures, and periods of the North American Southwest may consult Willey [56] for both general background and specific terminology.

Relevant to our discussion below are a few background remarks. Archaeologists distinguish four major prehistoric "cultural subareas" in the Southwest, which "correspond in large degree to natural environmental conditions" [56, p. 179]: Mogollon, Anasazi, Hohokam and Patayan (see Fig. 4). Present-day descendants are known for the last three groups (respectively, the Hopi and Zuñi

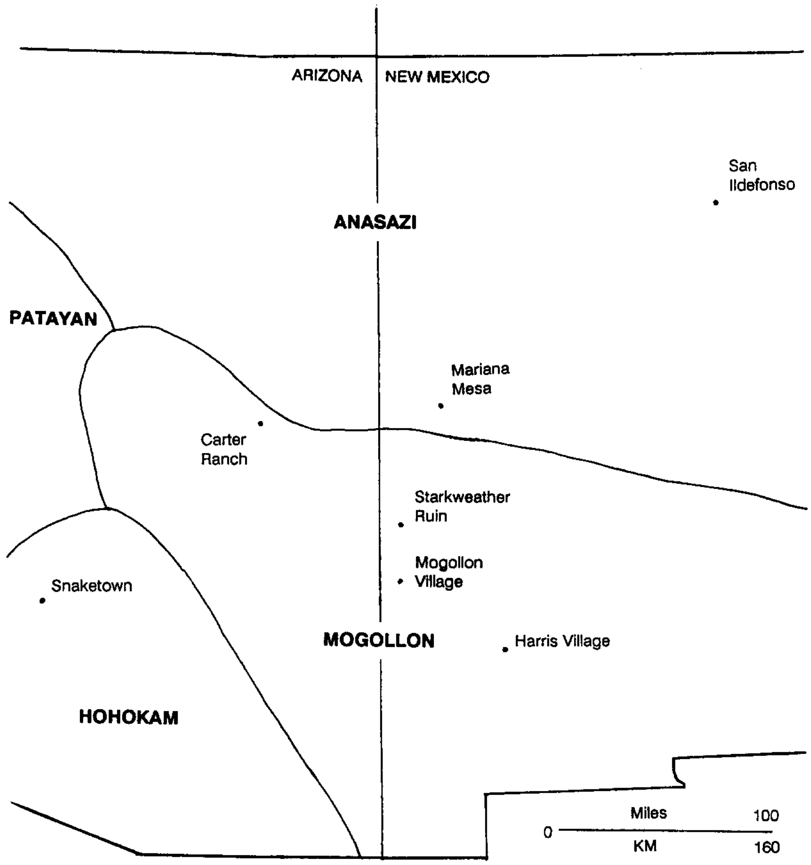

Fig. 4. Map of cultural subareas and sites referred to in text. (Adapted from *Emil W. Haury's Prehistory of the American Southwest* [Eds J. J. Reid and D. E. Doyel]. Univ. of Arizona Press, Tucson, Ariz. (1986).)

Pueblo Indians, the Pimas and Papagos, and Yuma-speaking Indians). It is unknown, however, what happened to the Mogollon people.

At the time of the excavation of Starkweather Ruin, the Mogollon culture had only just been identified by Emil Haury. In 1933 Haury had excavated Mogollon Village (named after the nearby Mogollon Mountains), a site close to Starkweather. It had yielded only pithouses, spanning the period from 550 to 900 A.D. [57]. The following year he had excavated a second site, Harris Village, in the Mimbres Valley some distance away, where he had found a similar sequence of remains spanning the same time range. Haury had found the sequence to be "fundamentally different from the Basketmaker-Pueblo (Anasazi) sequence to the north" [58, p. 298].

It was in order to obtain more information on the Mogollon culture that Nesbitt undertook to excavate at Starkweather. Nesbitt does not discuss in Ref. [59] how he chose that particular site, or even the origin of the name of the ruin [59, pp. 9, 10, 35–36, 78]:

> "Starkweather Ruin is situated . . . 3.5 miles west of Reserve, Catron County, New Mexico, in Section 3 Range 19 west, Township 7 south. The site occupies the west section of a small mesa which towers approximately eighty meters above the San Francisco river situated three miles to the northeast, and forty meters above Starkweather canyon directly to the south. The mesa is about 1850 meters above sea level. . . . The top of the mesa . . . measures approximately 400 meters east and west and 150 meters along the north–south axis. . . . Thirty-two houses were excavated, twelve pueblo surface houses and twenty pitrooms. . . . When the pueblo stone house builders came to Starkweather, the early pit dwellers had already moved elsewhere, and this later culture is not to be thought of as having evolved from that of the earlier occupants. Certain ceramic types do seem however to be derived from the pithouse period. . . . The ceramic collections from Starkweather village consist of the following materials:
>
> a. Approximately 12,000 sherds from the pitrooms and 5 complete and incomplete vessels.
> b. 302 complete specimens and several thousand sherds from the pueblo rooms and outlying burial ground."

Nesbitt classified the earlier pitroom culture as Mogollon, with two specimens of wood from one of the pithouses both giving bark dates of 927 A.D. He cautiously suggested that Mogollon occupation of the site may have lasted from 650 to 1100 A.D. He found the pithouse pottery as a whole to be "identical to that reported by Haury from the Mogollon and Harris village sites" [59, p. 114). His main conclusion was that the Mogollon culture is similar to the Basketmaker-Pueblo, with some Hohokam influence, in opposition to Haury, who had postulated the Mogollon as a third major cultural province. Nesbitt dated the first Mogollon settlements in New Mexico to 700–900 A.D., later than Haury. He also concluded that the Mogollon, Hohokam and Pueblo cultures derived certain ceramic types from the same source but that there was no evidence that the Mogollon was derived from any specific eastern group. For early reaction to Nesbitt's report, see Colton [60] and Kidder [61].

In the later 1930s tree-ring dating (and subsequently, radiocarbon dating) confirmed Haury's view of the Mogollon as a separate cultural area. Nesbitt, however, was right in arguing for cultural continuity rather than people moving in and out [62].

Our concern is not with the Mogollon but with the later Pueblo pottery found at Starkweather, which—unlike the browm Mogollon pottery—features painted designs. There are two such types: Tularosa black-on-white, dating from the Pueblo III horizon (1050–1300 A.D.) and the "earlier related type, probably ancestral" [59, p. 93] Reserve black-on-white from Pueblo II horizon (900–1100 A.D.).

COMMENTARY AND ANALYSIS

Table 1 summarizes observations made on the Starkweather pottery. The motif numbers given are keyed to Fig. 5.

Table 2 cross-classifies the data of Table 1 by type of pottery and symmetry of design. No expert was available to classify pots not otherwise identified; however, all but two of the untyped pots are manifestly either Reserve or Tularosa black-on-white.

The data of Washburn [10] included only one Reserve pot design but 69 Tularosa pot designs, compared with the 18 Tularosa, 36 Reserve, and 26 untyped designs of Starkweather. She distinguished between *symmetry of structure* and *symmetry of whole design* (pp. 18–19). Using the former appears to be close to the practice adopted by the present author. This author recorded (see Table 3) 12 designs with an asterisk, denoting that the pattern was slightly marred, and 11 more as possessing no symmetry.

The Tularosa-producing Starkweather site and the Mariana Mesa sites investigated by Washburn are contemporaneous Pueblo III sites of the eleventh and twelth centuries, roughly 60 miles apart. Tularosa black-on-white occurs at both locations. Other types of decorated pottery occur at Mariana Mesa but not at Starkweather, and the "probably ancestral" Reserve pottery of Starkweather is all but completely absent at Mariana Mesa. Washburn concluded (p. 172) that the distributions of pattern classes from the several sites were similar. The small number of Tularosa vessels at Starkweather, together with the differences in classification practices, does not afford close comparison with her data. However, the Starkweather data as a whole bear out the same themes, while offering some individuality.

The predominant symmetry pattern class at Starkweather is *p112*, whose characteristic element is a half-turn. Only two of the 17 bichromatic strip patterns occur, *p'112* and *p112'*, and both of these prominently feature the half-turn. Three of the monocolor strip patterns, *p1a1, p1m1,* and *pmm2* scarcely occur at all. *Pueblo art at Starkweather strongly preferred the symmetry of the half-turn; made some use of vertical mirror lines; and almost entirely avoided horizontal mirror lines and glide reflections.* Although Washburn's data included a larger proportion *pmm2* patterns, all of these "vanished" under her stricter classification by design symmetry. She appears to have observed a greater proportion of two-dimensional patterns, including some bichromatic ones. The avoidance of bilateral symmetry was a consistent feature of Western Hemisphere pottery; such symmetry was prominent only in Cochle, Panama [63].

The preference for half-turns and avoidance of glide reflection would be easy to explain if the

Table 1. Data on pottery

Vessel	Form	Type	Plate	Neck (rim) Elements	Neck (rim) Pattern	Neck (rim) Motif	Second neck (or rim) Elements	Second neck (or rim) Pattern	Second neck (or rim) Motif
21000	Bowl		—	7	*pm11*	30	33	*pm11*	30
22385	Eccentric pitcher	R	31B	9	*p112*	13			
22386	Pitcher	T	33G	4	*p112*	10			
22387	Pitcher	T	—	7+	*p112**	Other			
22388	Bowl		—	5?	Can't tell				
22389	Bowl		—	4	Can't tell				
22391	Bowl		—	4	*p112**	36,46			
22392	Bowl		—	2	*c2*	52			
22394	Bowl	R	29C	10	*p112*	52	7	*p112*	52
22395	Jar		—	5	*pma2**	71	4	*p111*	37
22396	Pitcher	R	31F	5	*pm11*	5			
22397	Pitcher	R	31I	4	*p111*	36			
22398	Pitcher	T	33F	17	*pm11**	25			
22399	Pitcher	R	30B	6	*p112′*	19			
22401	Bowl	R	29E	2	*c2*	7,53			
22401	Pitcher	R	31H	18	*p112*	tick			
22401/2	Bowl	T	32B	4	*p112′**	18			
22403	Bowl	T	32A	4	*p111*	Other			
22403	Pitcher	R	31C	4	*p112*	36R			
22409	Eccentric olla	R	31G	5	None	Cross	6	None	6,41
22411	Pitcher		—	3	*p112**	73			
22411/3	Pitcher	T	33D	10	*pmm2*	11	4 rows of 41	*cmm*	39,10
22417	Ladle		—	9	*p2*	10			
22418	Pitcher	T	33A	5	*p112*	36R			
22419	Hanging vessel		—						
22422	Pitcher		—	?	*pma2*	Line			
22423	Bowl	T	32D	4	*p112′**	19			
22424	Bowl	R	29B	4	*p112*	other			
22424/2	Bowl	R	29F	7	*p111*	other	5	*p111*	Other
22425	Pitcher	R	31E	4	*pm11*	39			
22425	Pitcher	R	30C	6	*p112*	36			
22425/5	Jar		—	5	None				
22427	Pitcher	R	30A	6	*pma2*	Line			
22434/3	Pitcher		33B		None				
22435	Jar		—		Can't tell				
22475	Pitcher	R	30D	6 rows of 48	*cmm*	39			
22476	Ladle	R	29D	2	*c2*	33,67			
22479	Bowl	Red-on-gray			Can't tell				
22479/1	Bowl	Red-on-buff		?	*p112*	52			

Not available for direct examination were the Starkweather vessels of Nesbitt's Plates 31A, 31D, 32C, 32E, 33C and 33E. Plate 32E. 22417, 22418 (2 items) and 22426.

Vessel—Logan Museum catalog accession number, which may cover several objects of the same provenience. Form—determined by author's by Nesbitt plate or Logan Museum catalog: left blank if neither of these classifies. T for Tularosa black-on-white; R for reserve instances of the basic motif. Pattern—An asterisk denotes deviation from exact symmetry. *c2, c4*: point symmetry pattern classes. *p111*, pattern classes (see Fig. 2). *p2′*: bichromatic periodic pattern class. Motif—Correlated to illustration numbers in Fig. 263, p. 167 of for bowls are entered in the data table in the same columns as for necks for pitchers. Patterns that covered the entire inside surface of

potter had executed part of the design, then turned the pot over, and repeated the same design while turning the pot in the same direction as before. The sense of motion, conveyed from the motion of the pot to the dynamic of the design, would favor half-turns and tend to work against the employment of horizontal mirror lines. The conjectured method, however, would not absolutely preclude horizontal mirror reflection, which would be easy to execute; so its absence may also be attributable to Pueblo esthetic taste.

The slight, perhaps deliberate, marring of the symmetry of some designs may be a manifestation of a cultural norm that artifacts should be left incomplete or imperfect, because completion is a bad omen, possibly associated with death [63]. A similar norm affected other American cultures and is from Starkweather Ruin

Body			Handle			
Elements	Pattern	Motif	Elements	Pattern	Motif	Remarks
22?	*pma2*	Other	18	*pm11*	30	"Starkweather?"
	None					3-bulb shape
5	*p112*	6		None		
5	*p112′**	16				Missing double (?) handle; right-angle jump in border; "probably Starkweather"
						Restored
						Restored
						Both patterns the same
7	*c2*	53,73				Additional symmetry element of curved vs jagged; no handle
3	*p112*	36,36R		None		
4	*p112*	Other	6	None	Lines	
3	*p′112**	52,53	3	*p112*	Line	
7	*p112′*	16				Body design continues across handle
7	*p1m1**	Lozenge, 73R	6	None	Line	
5	*p112*	36	2	*c2*	73	Body, Neck patterns reverses of each other
5	*p112**	19,46R, 47R				Nesbitt groups it in plate with pitchers
3	*p112′*	16				Handle missing
6	*p112**	5,23,46	4	*p111*	49,50R	Double handle, crossed over; checkerboard on second neck
3	*p111*	52,53		None		Figure handle (see Nesbitt p. 95).
5	*p112*	56,73				Most of top missing; lug on one side, other side broken away; frets on design
4	*p112*	Line, 29	?	*pma2*		
						Both patterns the same
5 bands: 10, 14, 16, 14, 11	*p112*	13				
3 bands of 6	*p112*	56,64	4	*c4*	51	
4	None	Spiral				Repaired; called "vase" in card catalog
5	*pmm2*	Cross, line	5	*pm11*	Line	
50	*p2*	61				
	p2′	16				Most of bottom missing; called "olla" in card catalog
7	*p112′*	16				Coiled handle; checker-board on neck
						20% fragment
						20% fragment

Plate 32E depicts the item with catalog number 22393; the other plates appear to correspond, not necessarily respectively, to item 22415,

consistent subjective classification. Agrees with Nesbitt plates and Logan Museum catalog card, except where noted. Type—as determined black-on-white. Plate—in Nesbitt [59]. Elements—number as counted by a naive observer, so that an element may consist of one or more *pm11, pma2, p1m1, pmm2:* strip pattern classes (see Fig. 1). *p′112, p112′*: bichromatic strip pattern classes (see Fig. 1). *cmm, p2:* periodic Washburn [10], reproduced here as Fig. 5. An R indicates the motif occurs in the opposite handedness than shown there. Strip patterns the bowl are entered in the data table under the heading for body.

a marked feature of Islamic art ("only Allah is perfect"—see [39, pp. 137–151] for an illustration of deliberate defects in the context of magic squares and their symmetries).

The presence at Starkweather of the (presumably) older Reserve pottery, together with the presence of Tularosa pottery at both Starkweather and Mariana Mesa, suggests the possibility of diffusion of style proceeding from one location to the other. We are mindful, however, of the cautions of Deetz and Detlefsen [64], who demonstrated the "Doppler effect" in archaeology: inferred rates of diffusion of styles can vary greatly with directionality of site sampling. We offer no speculation about connections between Mariana Mesa and Starkweather Ruin, or indeed any assertions about origins or spread of the design styles found at Starkweather. We know only that Pueblo people came

Table 2. Starkweather data on pattern classes tabulated by pottery type and location of pattern, with number of vessels analyzed indicated

	Tularosa			$N = 8$	Reserve			$N = 16$	Untyped			$N = 15$	All Combined			$N = 39$
	Neck	Body	Handle	Total	Neck	Body	Handle	Total	Neck	Body	Handle	Total	Neck	Body	Handle	Total
c2					2		1	3	1	1		2	3	1	1	5
c4							1	1							1	1
p111	1	1	1	3	3			3	1			1	5	1	1	7
p1a1														0		
pm11	1			1	2		1	3	2		1	3	5		2	7
p112	3	2	1	6	7	6		13	3	2		5	13	10	1	24
pma2					1			1	2	1	1	4	3	1	1	5
p1m1						1		1						1		1
pmm2	1			1		1		1					1	1		2
p'112		1		1										1		1
p112'	2	1		3	1	2		3		1		1	3	4		7
p2									1	1		2	1	1		2
cmm	1			1	1			1					2			2
p2'										1		1		1		1
None			2	2	2	1	3	6	2	1		3	4	2	5	11
Can't tell									4			4	4			4
Totals	9	5	4	18	19	11	6	36	16	8	2	26	44	24	12	80

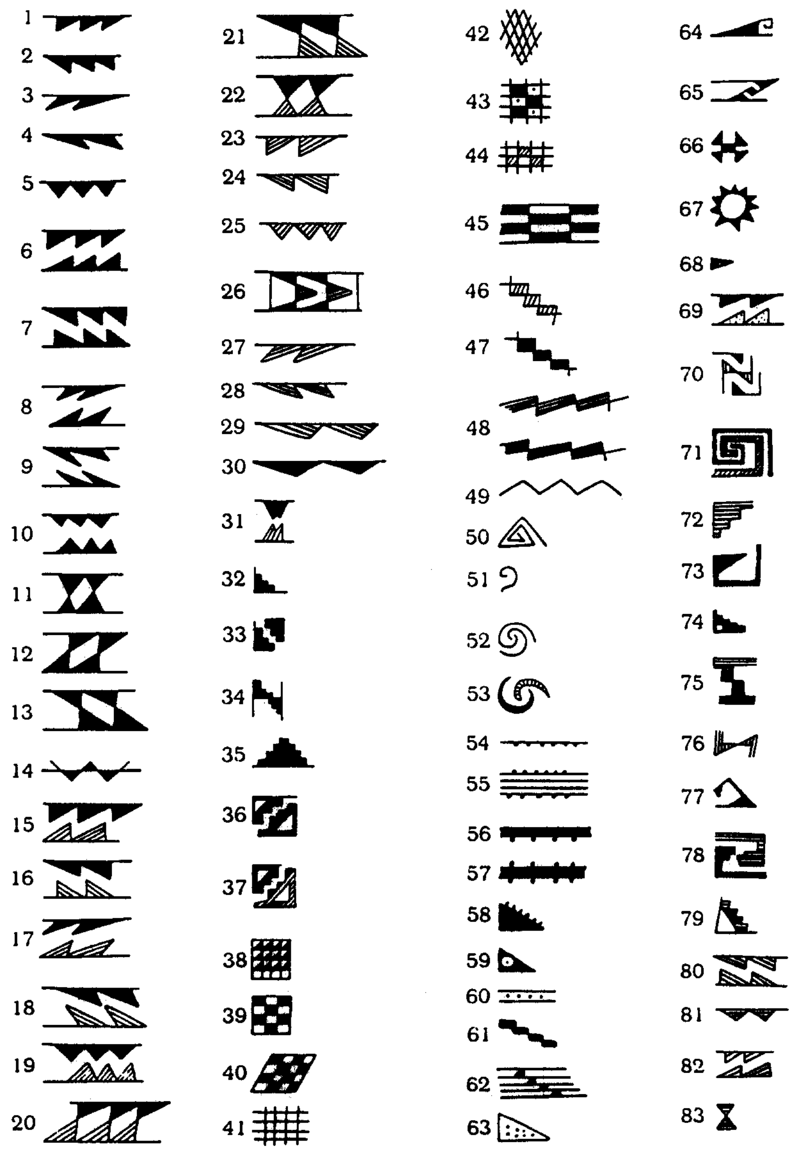

Fig. 5. Design elements. (Reproduced from Washburn [11, Fig. 263, p. 167].)

to live at Starkweather, their descendants dwelt there, and after a long occupation the inhabitants left and the site was abandoned.

We are not able to provide any sequencing of the pottery and designs found there, except to infer with Nesbitt that the cruder Reserve black-on-white preceded the Tularosa black-on-white.

Painstakingly-thorough analysis might permit the drawing of further conclusions about social organization of the inhabitants of the site. Longacre [65, 66] conducted a detailed "design element analysis" of 175 design elements on 6,000 sherds from a Pueblo settlement at the Carter Ranch Site, 80 miles from, and contemporaneous with, Starkweather. Since he worked from sherds, Longacre was not in a position to analyze symmetries of patterns of complete vessels, as we have done here. He hypothesized that:

> "If there were a system of localized matrilineal descent groups in the village, then ceramic manufacture and decoration would be learned and passed down within the lineage frame, it being presumed that the potters were female as they are today among the western Pueblos. Nonrandom preference for design attributes would reflect this social pattern." [65, p. 1454]

This hypothesis is still controversial. Longacre showed that design elements were associated with one of three groupings of rooms plus a kiva; and that "different types of pottery were associated with different rooms, and a set of stylistically distinct vessels was associated with each kiva and associated burials."

He concluded that the hypothesis of localized matrilineal groups was supported, that the household was the basic local unit, and that "communities made up of from one to three localized matrilineages were united through the mechanism of centralized ritual."

A study by Deetz [67] of the dynamics of stylistics change in the ceramics of the Arikara in South Dakota reached similar conclusions about that society, based on a detailed analysis of all features of the pottery and the same hypothesis:

> "Under a matrilocal rule of residence, reinforced by matrilineal descent, one might well expect a large degree of consistent patterning of design attributes, since the behavior patterns which produce these configurations would be passed from mothers to daughters [among the Arikara, most likely from grandmothers to granddaughters—see p. 97], and preserved by continuous manufacture in the same household. Furthermore, these attribute configurations would have a degree of mutual exclusion in a community, since each group of women would be responsible for a certain set of patterns differing more or less from those held by other similar groups." [67, pp. 2, 97]

Potters in market societies are known to be extremely conservative in style, because of their economic dependence on the market [68]. Longacre and Deetz suggested that matrilineal societies foster distinct pottery styles whose character is conserved not by market pressure but through tradition of the matrilineage. If they are right, then it should in theory be possible, with the help of symmetry analysis in addition to study of other ceramic attributes, to characterize the style or styles peculiar to each of the many Pueblo sites for which a large corpus of pottery is available.

One prevailing feature of the Starkweather collection points to the source of practical difficulty in even a limited version of such an undertaking: no two of the pots from Starkweather have the same design. Individual creativity may vary so much within a local style that the features of the style become impossible to discern accurately. The great variation among the Starkweather pots can be explained in part by features of Pueblo society. Many of the pots were found in burials, and the pot buried with a person in some sense must have partaken of that person's individuality. Moreover, a buried pot cannot be imitated (except from memory). Also, pots were made largely for family use, quite possibly one at a time as need arose. Another factor promoting individualization of designs within the familial tradition may have been need for a sense of achievement [69].

However, to the extent that the hypothesis of Longacre and Deetz holds true, the diffusion of similarities across Pueblo ceramics from widespread locations needs to be accounted for.

In this paper our purpose is more limited. Despite the differences noted above between Starkweather and Mariana Mesa ceramics, our major conclusion is that pattern analysis provides an easy comparison and confirmation of the similarities of the two sites, thus supporting Washburn's assertion:

> "It would appear that the use of symmetry classes to measure the similarities in design structure is a very consistent, objective procedure that can yield accurate, reproducible, and comparable results. The method clearly demonstrates the high degree of similarity of design structures among the Upper Gila area inhabitants and in this way confirms the postulated existence of an interacting community of potters." [10, p. 173]

Table 3. Comparison of pots by design symmetry class

	Washburn (Tularosa)		Starkweather			
	Design symmetry	Structure symmetry	Tularosa	Reserve	Untyped	Total
c1	1	1	0	0	0	0
c2	2	3	0	3	2	5
c4	0	1	0	1	0	1
d4	1	0	0	0	0	0
?	1	0	0	0	0	0
		5	0	4	0	6
p111	29	12	3	3	1	7
p1a1	0	0	0	0	0	0
pm11	3	7	1	3	3	7
p112	14	21	9	16	6	32
pma2	0	4	2	1	4	5
p1m1	1	0	0	1	0	1
pmm2	0	9	1	1	0	2
		53	16	25	14	54
p′112	0	0	1	0	0	1
p112′	6	(included above under *p112*)	3	3	1	7
p1		1	0	0	0	0
p2		5	0	0	2	2
pgg		3	0	0	0	0
pmg		2	0	0	0	0
cmm			1	1	0	2
p2′			0	0	1	1
		11	1	1	3	5
None		0	1	6	3	11
		0	1	6	3	11
Can't tell		0	0	0	4	4
		0	0	0	4	4
		69	18	36	26	80

THE VALUE AND MEANING OF SYMMETRY ANALYSIS

Of what significance is the work of this paper? For mathematicians, Pueblo pottery designs serve as realizations of abstract symmetry groups and offer their students practice in identification of symmetry elements and patterns. For anthropologists, pattern analysis represents a *new tool for identification and differentiation of patterned artifacts from closely-related cultures*. Enumeration of patterns employed by a culture, and comparison with those not employed, may yield insight into the esthetic sense and design process of the artisan. Consideration of the "geometric coercion" [63] imposed by the limited number of pattern classes casts a different light on the unlikelihood of the same pattern appearing on artifacts of two cultures, such as Valdivia in Ecuador and Middle Jomon in Japan [70]. For anyone, knowledge of the patterns affords an additional mode of appreciation of the artwork.

Does this style of analysis have anything to offer to the Native American who may be a descendant of the artisan? Are we uncovering or discerning Native American mathematics, or merely viewing sacred cultural remains through the eyes of a foreign and secular technology? Was the mathematics in the mind of the potter, or are we imposing it? Undoubtedly the symmetries are present in the pottery designs. The psychological difficulty in recognizing the devising of pottery designs as a form of mathematics is the preconception that there is only one true style of geometry: the written deduction of Euclid, subsequently "purified" by Hilbert to an axiomatic system completely devoid of figures and diagrams. The relevant analogue of this conception is the framework of abstract group theory in which it can be proved (with some difficulty!) that there are exactly seven symmetry groups of strip patterns, 17 of periodic patterns, and so on. Consider, though, the first investigators who tried to enumerate the 230 monocolor space groups. They did not proceed by rigorous logical group-theoretic reasoning; on the contrary, each of several individuals came up with incomplete enumeration in which some groups were listed twice. Comparisons and painful correction eventually led to a complete and correct list [3, pp. 132–133]. Were these crystallographers doing mathematics? Yes, they were, of a mixed inductive–deductive sort. So too were the Pueblo potters, though we have no narrative, only the pots, to stand as the record of their reasoning.

As a more current example of inductive mathematics we may cite the continuing investigation of what types of convex pentagons can tile the plane—that is, cover the plane with same-size, same-shape replicas without gaps or overlaps. One mathematician pronounced the subject closed and announced there were only eight types. A number of years later the topic was written up in a popular science magazine, and the article provoked one amateur to discover a ninth type; and a second amateur—a housewife with no formal education in mathematics beyond high school "general mathematics" 36 years earlier—then found five more over the next two years. A decade has passed, and still no one knows if the list is complete. The full fascinating story is related in Schattschneider [71]. She notes (p.166):

> "The mind and spirit are the forte of all such amateurs—the intense spirit of inquiry and the keen perception of all they encounter. No formal education provides these gifts. Mere lack of a mathematical degree separates these 'amateurs' from the 'professional'. Yet their dauntless curiosity and ingenious methods make them true mathematicians."

CONCLUSION

This paper has followed the practice of Washburn [10] in examining actual pottery vessels. This practice is important, because a photograph of one side of a vessel may be misleading in tending to indicate a degree of symmetry not borne out by the other side of the vessel. We have, however, concurred with Crowe and Washburn [5] in our scheme for classifying bichromatic patterns, rather than following the counterchange scheme of Washburn. The materials examined are related to those studied by Washburn, and much different from the ones studied by Zaslow and Dittert [43] (Hohokam culture, characterized by two-dimensional patterns), by Crowe and Washburn [52] (nineteenth century San Ildefonso culture, characterized by great variety and abundance of bichromatic patterns), and by Shepard [30] (who concentrated on identifying monochromatic band patterns on assorted Pueblo pottery).

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